

Universal scaling in BCS superconductivity in two dimensions in non- s waves

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Abstract

The solutions of a renormalized BCS model are studied in two space dimensions in s , p and d waves for finite-range separable potentials. The gap parameter, the critical temperature T_c , the coherence length ξ and the jump in specific heat at T_c as a function of zero-temperature condensation energy exhibit universal scalings. In the weak-coupling limit, the present model yields a small ξ and large T_c appropriate to those for high- T_c cuprates. The specific heat, penetration depth and thermal conductivity as a function of temperature show universal scaling in p and d waves.

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1 Introduction

At low temperature and in the weak coupling limit, a collection of weakly interacting electrons, spontaneously form large overlapping Cooper pairs [1] leading to the microscopic Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity [2, 3]. For usual superconductors, the s -wave BCS theory yields $\xi k_F \sim 1000$ in agreement with experiments, where ξ is the coherence length and k_F the Fermi momentum. There has been renewed interest in this problem with the discovery of high- T_c superconductors. The high- T_c materials have a small ξ : $\xi k_F \sim 10$ [4, 5]. In spite of much effort, the normal state of the high- T_c superconductors has not been satisfactorily understood [6]. There are controversies about the appropriate microscopic hamiltonian, pairing mechanism, and gap parameter [4, 5, 7].

Many high- T_c superconductors have a conducting structure similar to a two-dimensional layer of carriers [4, 7, 8], which suggests the use of two-dimensional models. Moreover, there are evidences that the high- T_c cuprates have singlet d -wave Cooper pairs and the gap parameter has $d_{x^2-y^2}$ symmetry in two dimensions [7]. Recent measurements of penetration depth $\lambda(T)$ [9] and superconducting specific heat at different temperatures T [10] and related theoretical analyses [11, 12] also support this point of view. According to the isotropic s -wave BCS theory,

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as $T \rightarrow 0$, both observables exhibit exponential dependence on T [3, 12]. The experimental power-law dependence of these observables on T can be explained by considering anisotropic gap parameter with node(s) on the Fermi surface in higher partial wave [11, 12]. It seems that some properties of the high- T_c cuprates can be explained by the d -wave BCS equation in two dimensions for weak coupling. Yet there has been no systematic study of the BCS equation in non- s waves in two dimensions.

The BCS theory considers N electrons of spacing L , interacting via a weak potential of short range r_0 such that $r_0 \ll L$ and $r_0 \ll R$ where R is the pair radius. When suitably scaled, most properties of the system should be insensitive to the details of the potential and be universal functions of the dimensionless variable L/R [13]. The usual BCS treatment [2] employs a phonon-induced two-electron potential of moderate range. In this paper we study the weak-coupling BCS problem in two dimensions for s , p , and d waves with two objectives in mind. The first objective is to identify the universal nature of the solution and its relation to high- T_c superconductors. The second objective is to find out to what extent the universal nature of the solution is modified in the presence of realistic finite-range (nonlocal separable) potentials. Instead of solving the BCS equation on the lattice with appropriate symmetry, we solved the equations in the continuum. This procedure should suffice for present objectives.

In place of the phonon-induced BCS model we employ a renormalized BCS model with separable potential which has certain advantage. The renormalized BCS model leads to convergent result even in the absence of potential form factors or momentum/energy cut off, as required in the standard BCS model. The original BCS model yields a linear correlation between T_c and T_D , where T_D is the Debye temperature. This correlation was fundamental in explaining the observed isotope effect in conventional superconductors. The high- T_c materials exhibit an anomalously negligible isotope effect and a linear correlation between T_c and T_F , where T_F is the Fermi temperature. This suggests a different interaction for superconductivity in the high- T_c materials. The present renormalized model yields a linear scaling between T_c and T_F . Because of this scaling with T_F , unlike in the phonon-induced BCS model, the present T_c can be large and appropriate for the high- T_c cuprates in the weak coupling region. The present model also produces an appropriate T_c/T_F ratio and a small ξk_F in the weak coupling region in accord with recent experiments [8] on high- T_c cuprates. In spite of these we are aware that there are controversies in the description of the high- T_c materials, for example, the microscopic hamiltonian and the pairing mechanism. Also, the normal state away from T_c seems to be very different from a standard Fermi liquid. However, we find that there are certain characteristics of these high- T_c materials which can be studied within the present renormalized mean-field BCS model based on the standard Fermi liquid theory.

Previously, there have been studies of this problem in two dimensions in terms of two-body binding in vacuum and Cooper pair binding employing short- and zero-range potentials [4, 5, 14]. Such studies have not fully revealed the universal nature of the solution. In the present paper we employ the zero-temperature condensation energy per particle, ΔU , of the BCS condensate as the reference variable for studying the BCS problem. As the condensation energy increases, one passes from the weak to medium coupling. Also, ΔU is a physical observable and is the appropriate reference variable as we shall see. In this paper we calculate the zero-temperature gap parameter $\Delta(0)$, the critical temperature T_c and the specific heat per particle $C_s(T_c)$ in all partial waves (denoted by angular momentum l) and the zero-temperature pair size in s wave. We find that these observables obey robust universal scaling as functions of ΔU valid

over several decades in the weak to medium coupling region independent of the potential range parameter.

Here, we also calculate the temperature dependence of different quantities, such as, the gap parameter $\Delta(T)$, $C_s(T)$, $\lambda(T)$ and thermal conductivity $K(T)$ for $T < T_c$. Of these, the T dependence of $C_s(T)$, $\lambda(T)$ and $K(T)$ are specially interesting. For isotropic s wave, the BCS theory yields exponential dependence on temperature as $T \rightarrow 0$ for these observables independent of space dimension [3, 12]. The observed power-law dependence of some these observables can be explained with anisotropic gap parameter in non- s waves with node on the Fermi surface. We find universal power-law dependence in both cases for non- s waves independent of the range of potential. For $l \neq 0$ we find $C_s(T) \approx 2C_n(T_c)(T/T_c)^2$ and $K_s(T) \approx K_n(T)(T/T_c)^{1.2}$ valid for almost the entire range of temperature. The suffix n and s refer to normal and superconducting states, respectively. Similar dependence was predicted from an analysis of experimental data [10] as well as from a calculation based on Eliashberg equation [11]. In order to detect the anisotropy in $\Delta(T)$ it is appropriate to consider the function $\Delta\lambda \equiv (\lambda(T) - \lambda(0))/\lambda(0)$ [12]. We find that for small T it behaves as $\Delta\lambda \sim (T/T_c)^{1.3}$. Similar power-law dependence was conjectured before [12].

From the weak-coupling BCS equation we establish the following relations analytically: $2\Delta(0)/T_c \approx 3.528$ (3.026), $\Delta(0) = 2\sqrt{\Delta U} (2\sqrt{\Delta U})$, $T_c \approx 1.134\sqrt{\Delta U} (1.322\sqrt{\Delta U})$, $\Delta C/C_n(T_c) \approx 1.43$ (1.05), $C_s(T_c) \approx 9.065\sqrt{\Delta U} (8.915\sqrt{\Delta U})$, $\Delta U/U_n(T_c) \approx 0.473$ (0.348), for $l = 0$ ($\neq 0$). Here ΔC is the jump in specific heat at $T = T_c$, $C(U)$ is the specific heat (internal energy) per particle.

The plan of the paper is as follows. In section 2 we present the renormalized BCS model. In section 3 we present an account of our numerical study. Finally, in section 4 we present a brief summary of our findings.

2 Renormalized BCS model

The two-body problem, Cooper and BCS models all exhibit ultraviolet divergences for zero range potential and require regularization or renormalization to produce finite results [4]. The usual BCS model employs a cut-off regularization (Debye cut off) for obtaining finite observables. In momentum space, the standard BCS model interaction is taken to be constant for momenta between $2m(E_F - E_D)/\hbar^2$ and $2m(E_F + E_D)/\hbar^2$ and zero elsewhere with E_D (E_F) the Debye (Fermi) energy. This implies a moderate range of the interaction. This potential is a physically motivated phonon induced electron-electron one [2]. The present renormalized BCS equation may lead to well-defined solution without requiring a cut off even in the absence of potential form factors. The present model with finite-range potential also leads to a well-defined mathematical problem.

We consider a two-body system, each of mass m , in the center of mass frame [4]. The single (two) particle energy is given by $\epsilon_q = \hbar^2 q^2/2m$ ($2\epsilon_q$), where q is the wave number. We consider a purely attractive short-range separable potential in partial wave l :

$$V_{\mathbf{p}\mathbf{q}} = -V_0 c_l g_{pl} g_{ql} \cos(l\theta) \quad (1)$$

where θ is the angle between vectors \mathbf{p} and \mathbf{q} and $c_l = 1$ (2) for $l = 0$ ($\neq 0$) as in Ref. [15]. Here g_{pl} and g_{ql} are potential form factors and V_0 is the potential strength. The potential $V_{\mathbf{p}\mathbf{q}}$

is the effective electron-electron potential in the superconductor in the presence of lattice and other electrons. The Schrödinger equation in this case leads to the following condition for a two-particle bound state in vacuum of binding B_2 is

$$V_0^{-1} = c_l \sum_{\mathbf{q}} g_{ql}^2 \cos^2(l\theta) (B_2 + 2\epsilon_q)^{-1} \quad (2)$$

where θ is the angle of vector \mathbf{q} and $\epsilon_q = \hbar^2 q^2 / 2m$ with m the mass. In order to simplify the notation, the trivial l dependence of many quantities, such as $V_{\mathbf{p}\mathbf{q}}$, B_2 , etc., is suppressed.

For attractive potential (2), two electrons on the top of the full Fermi sea at zero temperature shows pairing instability. Singlet (triplet) pairing is assumed to take place in even (odd) partial waves. The Cooper pair problem for two electrons above the filled Fermi sea for this potential is given by [1, 2] $V_0^{-1} = c_l \sum_{q>1} g_{ql}^2 \cos^2(l\theta) (2\epsilon_q - 2\hat{E})^{-1}$, with Cooper binding $B_c \equiv 2 - 2\hat{E}$. Using (2), the Cooper problem is written as

$$\begin{aligned} \sum_q g_{ql}^2 \cos^2(l\theta) (B_2 + 2\epsilon_q)^{-1} &= \sum_{q>1} g_{ql}^2 \cos^2(l\theta) (2\epsilon_q - 2\hat{E})^{-1} \\ &= 0. \end{aligned} \quad (3)$$

At a finite T , one has the following BCS and number equations

$$\Delta_{\mathbf{p}} = - \sum_{\mathbf{q}} V_{\mathbf{p}\mathbf{q}} \frac{\Delta_{\mathbf{q}}}{2E_{\mathbf{q}}} \tanh \frac{E_{\mathbf{q}}}{2T}, \quad (4)$$

$$N = \sum_{\mathbf{q}} \left[1 - \frac{\epsilon_q - \mu}{E_{\mathbf{q}}} \tanh \frac{E_{\mathbf{q}}}{2T} \right] \quad (5)$$

with $E_{\mathbf{q}} = [(\epsilon_q - \mu)^2 + |\Delta_{\mathbf{q}}|^2]^{1/2}$, where $\Delta_{\mathbf{q}}$ is the gap function. Unless the units of the variables are explicitly mentioned, in (2) – (5) and in the following all energy/momentum variables are expressed in units of E_F , such that $\mu \equiv \mu/E_F$, $T \equiv T/T_F$, $q \equiv q/k_F$, $E_{\mathbf{q}} \equiv E_{\mathbf{q}}/E_F$, etc, where μ is the chemical potential. Here $\Delta_{\mathbf{q}}$ has the following anisotropic form: $\Delta_{\mathbf{q}} \equiv g_{ql} \Delta_0 \sqrt{c_l} \cos(l\theta)$ where Δ_0 and g_{ql} are dimensionless. The usual BCS gap is defined by $\Delta(T) = g_{q(=1)l} \Delta_0$, which is the root-mean-square average of $\Delta_{\mathbf{q}}$ on the Fermi surface. Using the above form of $\Delta_{\mathbf{q}}$, the BCS equation (4) can be written as

$$\frac{1}{V_0} = c_l \sum_{\mathbf{q}} g_{ql}^2 \cos^2(l\theta) \frac{1}{2E_{\mathbf{q}}} \tanh \frac{E_{\mathbf{q}}}{2T}. \quad (6)$$

Now (2) and (6) lead to

$$\sum_{\mathbf{q}} g_{ql}^2 \cos^2(l\theta) \left[\frac{2}{2\epsilon_q + B_2} - \frac{1}{E_{\mathbf{q}}} \tanh \frac{E_{\mathbf{q}}}{2T} \right] = 0. \quad (7)$$

The summation is evaluated according to

$$\sum_{\mathbf{q}} \rightarrow \frac{N}{4\pi} \int d\epsilon_q d\theta \equiv \frac{N}{4\pi} \int_0^\infty d\epsilon_q \int_0^{2\pi} d\theta \quad (8)$$

where N is the number of electrons. As in the standard BCS model, we have a constant density of state in (8) but now with the generalization to include the angular dependence. With the help of (8), (5) and (7) can be explicitly written as

$$\int d\epsilon_q d\theta \left[1 - \frac{\epsilon_q - \mu}{E_{\mathbf{q}}} \tanh \frac{E_{\mathbf{q}}}{2T} \right] = 4\pi \quad (9)$$

$$\int d\epsilon_q d\theta g_{ql}^2 \cos^2(l\theta) \left[\frac{2}{2\epsilon_q + B_2} - \frac{1}{E_{\mathbf{q}}} \tanh \frac{E_{\mathbf{q}}}{2T} \right] = 0 \quad (10)$$

respectively. In terms of Cooper-pair binding Eq. (10) can be rewritten as

$$\int_0^{2\pi} d\theta \cos^2(l\theta) \cdot \left[\int_1^\infty d\epsilon_q \frac{g_{ql}^2}{\epsilon_q - \hat{E}} - \int_0^\infty d\epsilon_q \frac{g_{ql}^2}{E_{\mathbf{q}}} \tanh \frac{E_{\mathbf{q}}}{2T} \right] = 0 \quad (11)$$

Even in the absence of potential form factors $g_{pl} = 1$, (9), (10) and (11) are well defined without any energy/momentum cut-off, though each part of the integral in these equations diverges separately. This is why this model is termed renormalized. Potential (1) with $g_{pl} = 1$ is the zero-range delta-function potential. The standard BCS model uses essentially the above potential with energy/momentum cut-off for obtaining convergence. In renormalized equation (10) the usual energy/momentum cut-off and the potential strength V_0 have been eliminated in favor of the two-body binding B_2 . Recently, the role of renormalization in nonrelativistic quantum mechanics has been discussed [16].

Equation (11) has following analytic solutions for s -wave zero-range potential ($g_{q0} = 1$) in the weak coupling limit ($\mu = 1$). At $T = 0$, $\Delta(0) = \sqrt{2B_c}$. At $T = T_c$ ($\Delta = 0$), we have: $T_c = \exp(\gamma)\sqrt{2B_c}/\pi \approx 0.8\sqrt{B_c}$ where $\gamma = 0.57722\dots$. The standard BCS model yields in this case $T_c/T_D = \exp(\gamma)\sqrt{2B_c}/\pi\sqrt{T_D} \approx 0.8\sqrt{B_c/T_D}$ where T_D is the Debye temperature. To illustrate the advantage of the renormalized model let us consider a specific example with $T_D = 300$ K, $T_F = 3000$ K and $B_c = 10$ K. We take $B_c < 10$ K as defining the weak-coupling region. With $B_c = 10$ K, the standard BCS model yields $T_c = 44$ K, whereas the present renormalized model yields $T_c = 138$ K. Hence for same coupling, the renormalized model leads to an enhanced T_c . In the standard BCS model one has to have a much larger B_c , clearly outside the weak-coupling region, in order to have $T_c > 100$ K.

The universal ratio $2\Delta(0)/T_c = 2\pi/\exp(\gamma) \approx 3.528$ remains unchanged for s wave in three dimensions as well as for the trivial case of anisotropic pairing ($\sim \exp(il\theta)$) in two dimensions for $l \neq 0$ [14]. In Ref. [14] we essentially changed the potential form-factors without introducing any explicit angular dependence in the BCS and number equations and find that the universal nature of the solution was unchanged with such change. Hence we expect to extract certain universal properties of (10) analytically for $l \neq 0$ by employing the correct angular distribution and no potential form factors ($g_{ql} = 1$).

Next we study (10) analytically for weak coupling ($\mu = 1, \Delta \ll 1$) and for $g_{ql} = 1$. At $T = 0$, (10) can be integrated to yield

$$\int_0^{2\pi} d\theta \cos^2(l\theta) \ln \frac{[1 + \Delta^2(0)c_l \cos^2(l\theta)]^{1/2} - 1}{B_2} = 0,$$

which for $\Delta(0) \ll 1$ and $l \neq 0$ reduces to

$$\ln \frac{c_l \Delta^2(0)}{2B_2} = -\frac{1}{\pi} \int_0^{2\pi} d\theta \cos^2(l\theta) \ln \cos^2(l\theta) \equiv 0.3863,$$

or, $\Delta(0) \approx 1.213\sqrt{B_2}$. At $T = T_c$, we again have $T_c = \exp(\gamma)\sqrt{2B_2}/\pi$, so that we have the following new universal constant for $l \neq 0$: $2\Delta(0)/T_c \approx 3.026$.

The condensation energy per particle at $T = 0$ is defined by [3]

$$\Delta U \equiv |U_s - U_n| = \frac{1}{N} \sum_{\mathbf{q}(q < 1)} 2\zeta_q - \frac{1}{N} \sum_{\mathbf{q}} \left(\zeta_q - \frac{\zeta_q^2}{E_{\mathbf{q}}} - \frac{\Delta_{\mathbf{q}}^2}{2E_{\mathbf{q}}} \right)$$

where $\zeta_q = (\epsilon_q - \mu)$. This can be evaluated straightforwardly to lead to [3]

$$\Delta U = \frac{1}{8\pi} \int_0^{2\pi} c_l \Delta^2(0) \cos^2(l\theta) d\theta$$

which yields $\Delta(0) = 2\sqrt{\Delta U}$ for all l . Using the universal relation between $\Delta(0)$ and T_c given above one has $T_c \approx 1.134\sqrt{\Delta U}$ ($1.322\sqrt{\Delta U}$) for $l = 0$ ($l \neq 0$). For all l , $U_n(T_c) = \pi^2 T_c^2/6$, so that $\Delta U/U_n(T_c) \approx 0.473$ (0.348) for $l = 0$ ($\neq 0$).

The superconducting specific heat per particle is given by

$$C_s = \frac{2}{NT^2} \sum_{\mathbf{q}} f_{\mathbf{q}}(1 - f_{\mathbf{q}}) \left(E_{\mathbf{q}}^2 - \frac{1}{2} T \frac{d\Delta_{\mathbf{q}}^2}{dT} \right) \quad (12)$$

where $f_{\mathbf{q}} = 1/(1 + \exp(E_{\mathbf{q}}/T))$. The normal specific heat C_n is given by (12) with $\Delta_{\mathbf{q}} = 0$. The jump in specific heat per particle at $T = T_c$ ($\Delta(T_c) = 0$), $\Delta C \equiv [C_s - C_n]_{T_c}$ is given by [3]

$$\Delta C = -\frac{1}{NT_c} \sum_{\mathbf{q}} \left[f_{\mathbf{q}}(1 - f_{\mathbf{q}}) \frac{d\Delta_{\mathbf{q}}^2}{dT} \right]_{T_c}. \quad (13)$$

In the special case $g_{ql} = 1$, the radial integral in (13) can be evaluated as in Ref. [3] and we get

$$\Delta C = -\int c_l \frac{d\epsilon_q d\theta}{4\pi T_c} \left[f_{\mathbf{q}}(1 - f_{\mathbf{q}}) \frac{d\Delta^2(T)}{dT} \right]_{T_c} \cos^2(l\theta). \quad (14)$$

This leads to [3] $\Delta C = -(1/2)[d\Delta^2(T)/dT]_{T=T_c}$ for all l . In a systematic (numerical) study we find $\Delta(T)/\Delta(0) = B(1 - T/T_c)^{1/2}$ valid for $T \approx T_c$, with $B \approx 1.74$ (1.70) for $l = 0$ ($\neq 0$). Using the value of B and the universal ratio $2\Delta(0)/T_c$, we obtain $\Delta C/C_n(T_c) \approx 1.43$ (1.00) for $l = 0$ ($\neq 0$), where $C_n(T) = \pi^2 T/3$. Consequently, $C_s(T_c)/\sqrt{\Delta U} \approx 9.065$ (8.915) and $T_c C_n(T_c)/\Delta U \approx 4.229$ (5.749) for $l = 0$ ($\neq 0$).

The penetration depth λ is defined by [3]

$$\lambda^{-2}(T) = \lambda^{-2}(0) \left[1 - \frac{2}{NT} \sum_{\mathbf{q}} f_{\mathbf{q}}(1 - f_{\mathbf{q}}) \right]. \quad (15)$$

The thermal conductivity ratio $K_s(T)/K_n(T)$ is defined by [17]

$$\frac{K_s(T)}{K_n(T)} = \frac{\sum_{\mathbf{q}} \zeta_q E_{\mathbf{q}} f_{\mathbf{q}}(1 - f_{\mathbf{q}})}{\sum_{\mathbf{q}} \zeta_q^2 f_{\mathbf{q}}(1 - f_{\mathbf{q}})}. \quad (16)$$

The denominator of (16) essentially corresponds to the normal-state thermal conductivity with the BCS gap set equal to zero.

The dimensionless s -wave pair radius defined by $\xi^2 = \langle \psi_q | r^2 | \psi_q \rangle / \langle \psi_q | \psi_q \rangle$ with the pair wave function $\psi_q = g_{ql} \Delta / (2E_q)$ can be evaluated by using $r^2 = -\nabla_q^2$. In the weak coupling limit, the zero-range analytic result of Ref. [4] leads to $\xi^2 = 0.5 \Delta^{-2}(0) = 0.125 (\Delta U)^{-1}$.

3 Numerical Results

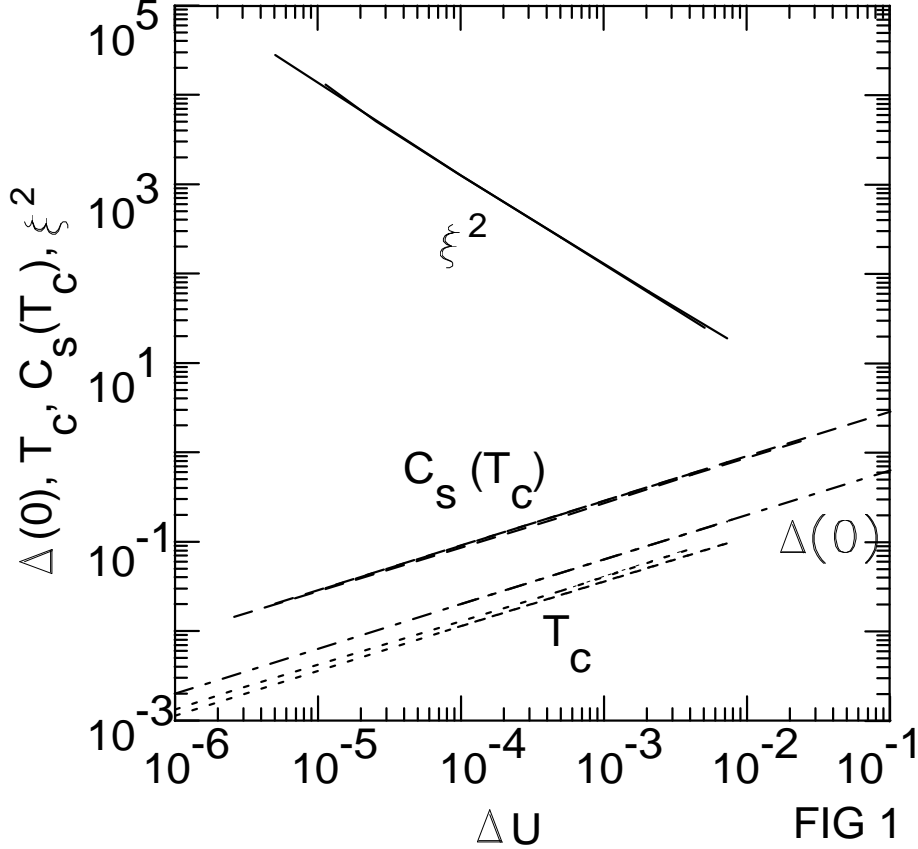


Fig. 1 The plot of specific heat $C_s(T_c)$ (dashed line), T_c (dotted line), gap parameter $\Delta(0)$ (dashed-dotted line) for s , p , and d waves and s -wave pair radius ξ^2 (solid line) versus zero-temperature condensation energy per particle ΔU for different potential parameters between $\alpha = 1$ to ∞ . For the first two variables there are two distinct lines; the upper one for p and d waves and the lower one for s wave.

We solve coupled equations (9) and (10) numerically using dimensionless form factors $g_{ql} = (\epsilon_q)^{l/2} [\alpha / (\epsilon_q + \alpha)]^{(l+1)/2}$ with the correct threshold behavior for small momenta, where α is the range parameter. Following Ref. [3] we calculate the dimensionless gap parameter $\Delta(0) = g_{q(=1)l} \Delta$, T_c , $C_s(T_c)$, the s -wave pair radius ξ^2 at $T = 0$ as well as $\Delta(T)$, $\lambda(T)$, and $C(T)$ for different coupling. In figure 1 we plot $\Delta(0)$, T_c , $C_s(T_c)$, and ξ^2 at $T = 0$ versus ΔU and find universal scalings. The calculations were repeated for different potential ranges α . We varied α from 1 to ∞ and found figure 1 to be insensitive to this variation in each partial wave. For p

and d waves (9) and (10) diverge for $\alpha \rightarrow \infty$ and calculations were performed for $\alpha = 1$ to 10. The increase in ΔU of figure 1 corresponds to an increase in coupling. We could express this change in coupling by a change in B_2 or in Cooper pair binding and plot the variables of figure 1 in terms of these bindings as in Ref. [14]. Then each value of range parameter leads to a distinct curve. However, if we express the variation in coupling by a variation of an observable of the superconductor, such as ΔU or T_c , universal potential-independent scalings are obtained. In each case the exponent and the prefactor of each scaling relation are in excellent agreement with the analytic relation obtained above without form factors.

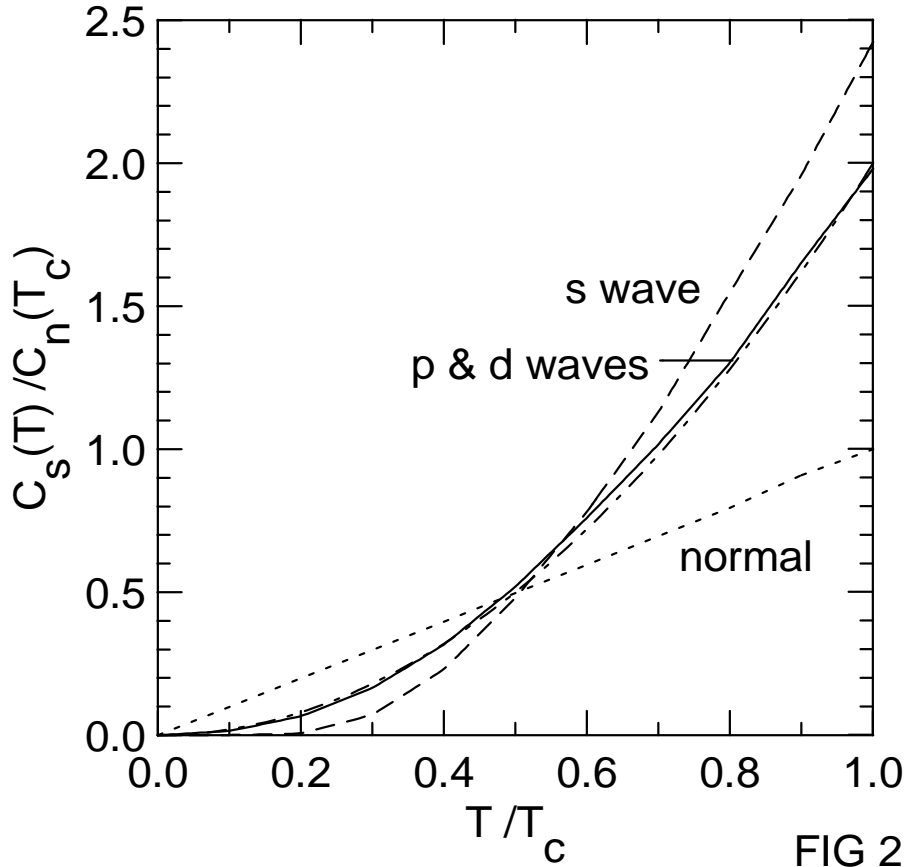


FIG 2
Fig. 2 Specific heat $C_s(T)/C_n(T_c)$ versus T/T_c for s (dashed line), p and d (solid line) waves and potential parameters between $\alpha = 1$ to ∞ . The analytic fit (dashed-dotted line) $C_s(T)/C_n(T_c) = 2(T/T_c)^2$, to p and d waves is also shown.

The T_c should not arbitrarily increase with coupling as figure 1 may imply. With increased coupling the electron pairs should form composite bosons which may undergo a phase transition under the action of a residual interaction. According to a numerical study this transition happens at a temperature of 0.1 [18]. This is why the T_c curve in figure 1 has been plotted till about $T_c = 0.1$. For a very large class of two-dimensional high- T_c materials, the T_c has been estimated to be about 0.05 [8], which corresponds, for $g_{ql} = 1$, to $B_2 = 0.004$. This small value of B_2 is in the weak coupling region where the universality of the present study should hold. The corresponding dimensionless pair radius ($\xi^2 \sim 80$) at $T_c = 0.05$, as obtained from figure 1, is in agreement with experimental analysis [8]. Hence, the present study is relevant for these high- T_c materials.

Next we studied the temperature dependence of $\Delta(T)$, $C_s(T)$, $\lambda(T)$ and $K(T)$ for $T < T_c$. For BCS superconductors, these observables have exponential dependence on T as $T \rightarrow 0$ [3, 12]. The two-dimensional high- T_c superconductors have a power-law dependence on T . In figures 2, 3, and 4 we plot $C_s(T)/C_n(T_c)$, $\Delta\lambda(T) \equiv (\lambda(T) - \lambda(0))/\lambda(0)$, and $K_s(T)/K_n(T)$ versus T/T_c , respectively. In these figures we find universal power-law dependence, essentially independent of potential range, for $l \neq 0$. We find $C_s(T) \approx 2C_n(T_c)(T/T_c)^2$, $K_s(T) \approx 2K_n(T)(T/T_c)^{1.2}$ for almost the entire temperature range and $\Delta\lambda(T)/\lambda(0) \sim (T/T_c)^{1.3}$ for small T/T_c . The T^2 dependence of $C_s(T)$ was found in a theoretical study of the Eliashberg equation [11] and in an analysis of experimental data [10]. The power-law dependence of $\lambda(T)$ on T was also conjectured before [12]. The gap function $\Delta(T)$ has essentially the same universal behavior as in s wave [3].

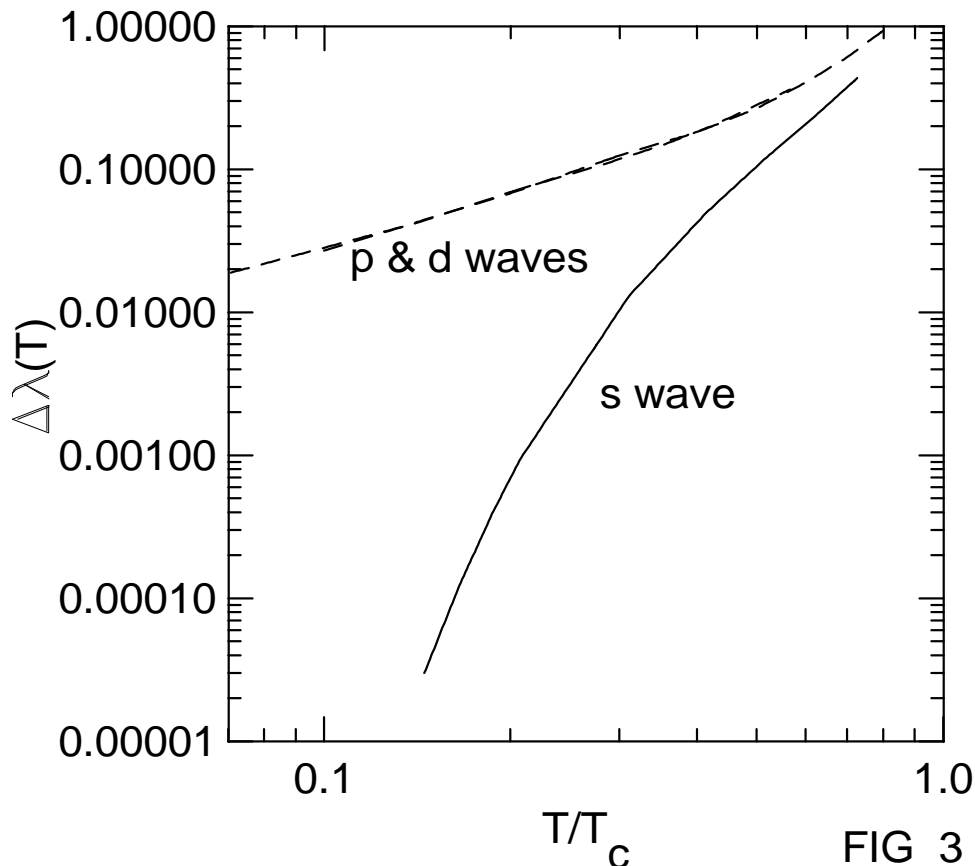


Fig. 3 Penetration depth $\Delta\lambda(T)$ versus T/T_c for s (dashed line), p and d (solid line) waves and potential parameters between $\alpha = 1$ to ∞ .

4 Summary

Scalings are established among $\Delta(0)$, T_c , $C_s(T_c)$, and ξ^2 , as function of ΔU , independent of potential range in s , p and d waves from a study of a renormalized BCS equation in two dimensions. The present renormalized model yields a large T_c and a small ξ appropriate for

some high- T_c superconductors. The T dependence of $\Delta\lambda(T)$, $C_s(T)$ and $K_s(T)$ below T_c in non- s waves show power-law scalings distinct to some high- T_c materials at low energies. No power-law T dependence is found in s wave for $\Delta\lambda(T)$ and $C_s(T)$. Calculations performed with $\cos(l\theta)$ and $\sin(l\theta)$ angular dependences yielded identical results. Though we have used a separable potential, in view of the universal nature of the study we do not believe the present conclusions to be so peculiar as to have no general validity. We have repeated the s wave calculations with local Yukawa potential and found the results to be independent of potential in the weak-coupling region. This is also in agreement with a suggestion by Leggett [13]. Although, there are controversies about a microscopic formulation of high- T_c superconductors, it seems that the two-dimensional d -wave BCS equation for weak coupling can be used to explain some of their universal scalings. A similar study of universality has recently been performed in three dimensions employing the renormalized BCS equation [19], which also leads to an enhanced T_c in the weak-coupling limit.

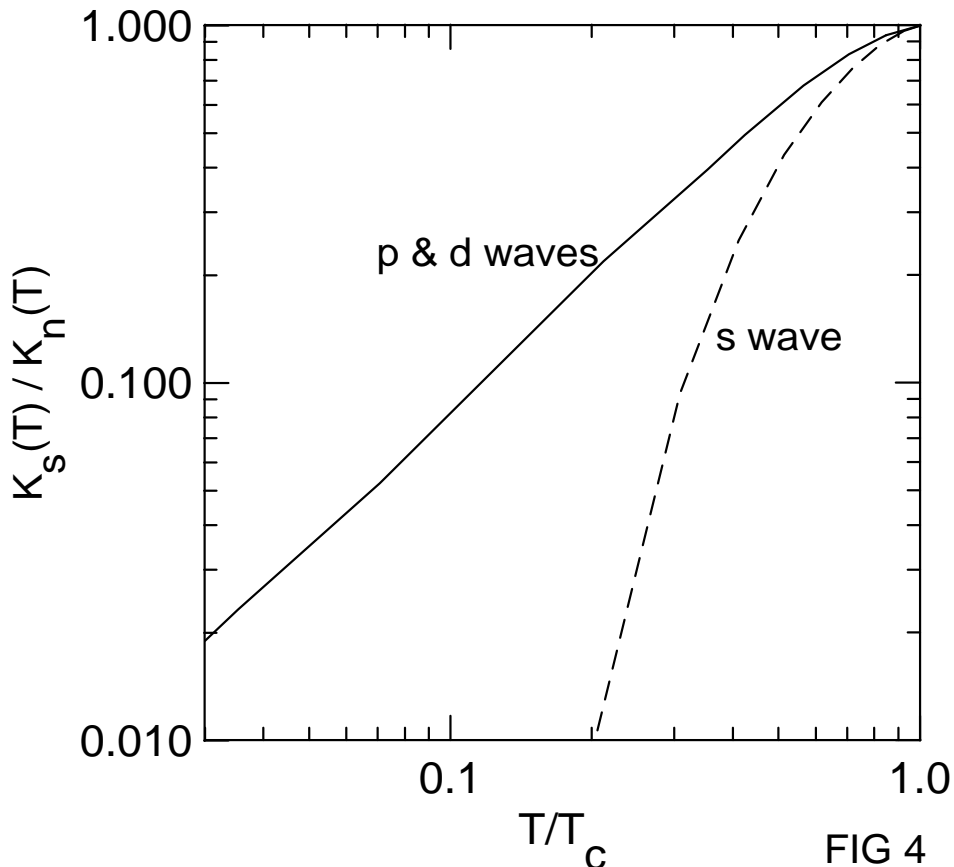


Fig. 4 Thermal conductivity ratio $K_s(T)/K_n(T)$ versus T/T_c for s (dashed line), p and d (solid line) waves and potential parameters between $\alpha = 1$ to ∞ .

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